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MA222 - Computational Linear Algebra
Problem Sheet - 3

Block Matrices and Algorithms

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1. Adapt strass so that it can handle square matrix multiplication of any order.
   Hint: If the "current" A has odd dimension, append a zero row and column.

2. Prove that if

$$A = \begin{bmatrix} A_{11} & \cdots & A_{1r} \\ \vdots & \ddots & \vdots \\ A_{q1} & \cdots & A_{qr} \end{bmatrix}$$

   is a blocking of the matrix A, then

$$A^T = \begin{bmatrix} A_{11}^T & \cdots & A_{q1}^T \\ \vdots & \ddots & \vdots \\ A_{1r}^T & \cdots & A_{qr}^T \end{bmatrix}.$$

3. Suppose n is even and define the following function from $\mathbb{R}^n$ to $\mathbb{R}$:

$$f(x) = x(1:2:n)^T x(2:n) = \sum_{i=1}^{n/2} x_{2i-1}x_{2i}$$

   (a) Show that if $x, y \in \mathbb{R}^n$ then

$$x^T y = \sum_{i=1}^{n/2} (x_{2i-1} + y_{2i})(x_{2i} + y_{2i-1}) - f(x) - f(y)$$

   (b) Now consider the n-by-n matrix multiplication $C = AB$. Give an algorithm for computing this product that requires $n^3/2$ multiplies once f is applied to the rows of A and the columns of B.

4. Prove Lemma 1.3.2 for general s.
   Hint. Set $p_T = p_1 + \cdots + p_{\gamma-1} \qquad \gamma = 1 : s+1$ and show that $c_{ij} = \sum_{\gamma=1}^{s} \sum_{k=p_\gamma+1}^{p_{\mu+1}} a_{ik}b_{kj}$.

5. Use Lemmas 1.3.1 and 1.3.2 to prove Theorem 1.3.3. In particular, set

$$A_\gamma = \begin{bmatrix} A_{1\gamma} \\ \vdots \\ A_{q\gamma} \end{bmatrix} \quad \text{and} \quad B_\gamma = \begin{bmatrix} B_{\gamma 1} & \cdots & B_{\gamma r} \end{bmatrix}$$

   and note from Lemma 1.3.2 that

$$C = \sum_{\gamma=1}^{s} A_\gamma B_\gamma.$$

   Now analyze each $A_\gamma B_\gamma$ with the help of Lemma 1.3.1.

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